Chord direction in asteroid occultations: Effect of Earth rotation and orientation combined with asteroid relative velocity

Frank Freestar8n

This report conveys why the chord measured as an asteroid occults a star depends on many parameters related to the observer's location on the rotating Earth. Several factors are at work: Earth rotation and tilt as seen by the asteroid, observer latitude and proximity to the Earth's limb (i.e. the altitude of the asteroid in the observer's sky), and relative velocity of the asteroid to the Earth.
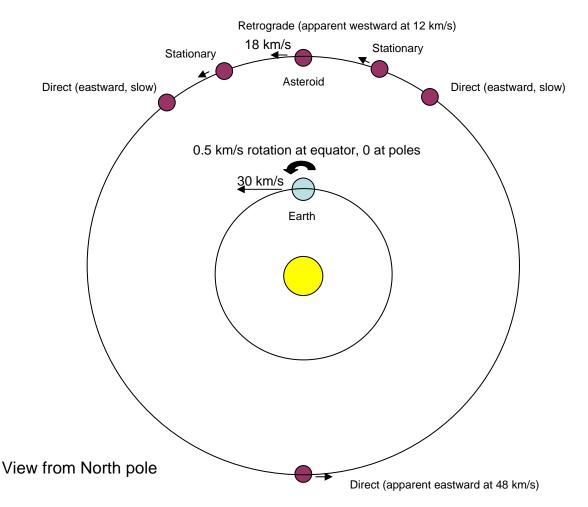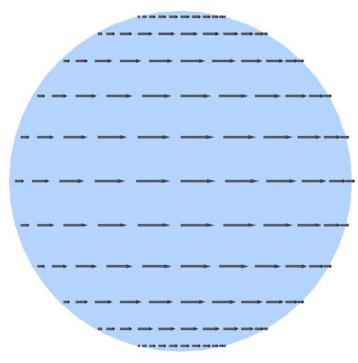

Figure 1. Basic geometry of orbits and rotation with associated velocities.

The figure above shows the orbital and rotational components that dominate the motion of an asteroid during an occultation. The Earth orbits counter-clockwise at 30 km/s, while farther away main-belt asteroids orbit in the same direction at about 18 km/s. At opposition the asteroid appears to be moving westward (retrograde) at 12 km/s relative to a stationary Earth. On the other side of the sun, the asteroid appears to move eastward at 48 km/s. Near opposition, approximately 54 degrees either side, the asteroid appears to stop its east-west motion because its relative motion is in the direction of Earth. On either side near that stationary point, the asteroid will appear to be moving slowly relative to the stars.

These motions determine the rate and direction of the asteroid as it crosses a star as seen from the center of the Earth, but for observers on the surface there is an additional rotational component that plays a role, particularly when the apparent motion of the asteroid is slow. The Earth's rotation causes an observer on the equator to move eastward at approximately 0.5 km/s. From the diagram above, this adds to the orbital velocity during the night, and subtracts during the day. The velocity decreases with cos(latitude) and is 0 at the poles.

Although this rotational velocity is small compared to the orbital velocities of the Earth and asteroid, it nonetheless can have a significant impact on an occultation chord, particularly when the asteroid's apparent motion against the stars is slow. A key factor is that the motion of an observer on the Earth depends on latitude, tilt of the Earth, and proximity to the limb. The combination of three-dimensional geometry with flattening projections makes this difficult to visualize without explicitly constructed figures, which is a key purpose of this write up.
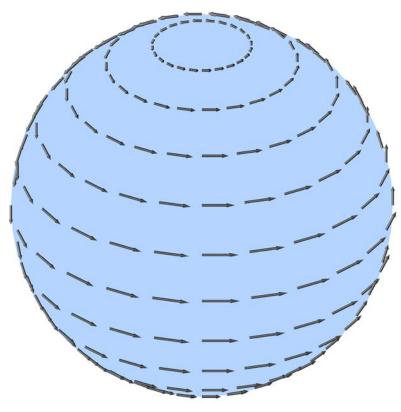
Figure 2 shows the variation of velocity due to Earth rotation as seen with no tilt of the Earth's axis. Note that the peak velocity is in the center, on the equator, at 0.5 km/s, but it decreases due to a combination of latitude (a true decrease in velocity) and with proximity to the limb, which is a projection effect. Note also that, due to projection, more observers will tend to be near the limb because more surface area is near the limb.
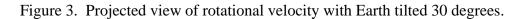

Earth tilt 0 degrees   Asteroid relative velocity 0.00 x, 0.00 y km/s

Figure 2. Apparent velocity across the surface of the rotating Earth. Note the combination of factors due to latitude and projection near the limb.

Figure 3 includes the effect of orientation, since the Earth is tilted on its axis and asteroids can be far from the ecliptic in their orbits. We now see a variation not only in speed, but in direction; near the limb on the equator, the velocity is up on one side and down on the other. Regardless of the asteroid's relative velocity, the projected velocity of the Earth's rotation will always contribute to the apparent motion of the asteroid during an occultation, and will alter the chord direction correspondingly.
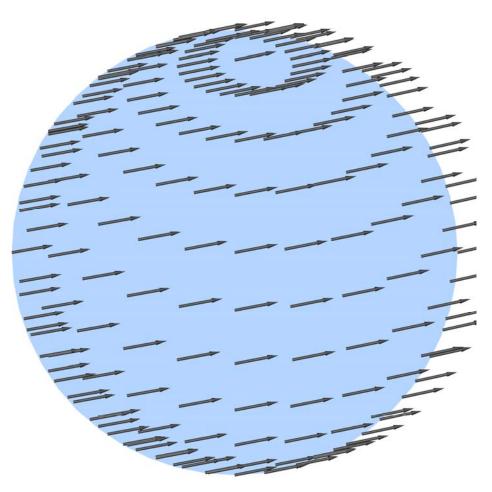
Earth tilt 30 degrees   Asteroid relative velocity 0.00 x, 0.00 y km/s

Figure 3. Projected view of rotational velocity with Earth tilted 30 degrees.

Although the relative velocity of asteroid to the Earth can be much greater than the 0.5 km/s rotational speed at the equator, when the asteroid appears to reverse direction in the sky, its apparent motion can become very small and dominated by motion perpendicular to the ecliptic. Occultations by asteroids moving slowly are correspondingly rare since they sweep out less of the sky over time and are less likely to intercept a star. Nonetheless, at any speed the rotation of the Earth alters the chord direction depending on the observer's location, and for slow asteroids this effect can be dramatically variable across the Earth's surface.
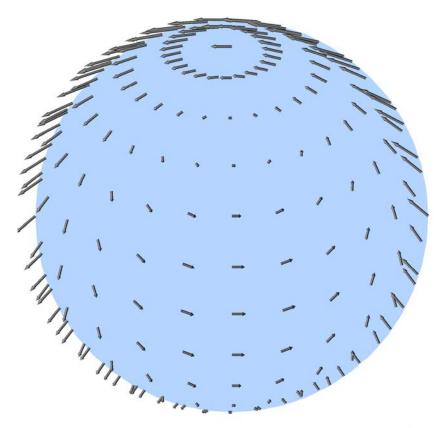
Figure 4 shows a view of the Earth from an asteroid moving with a relative velocity of 10 km/s in the x-direction and 2 km/s in the y-direction. Since the velocity is much greater than 0.5 km/s, all vectors appear nearly parallel, but in fact have slight variation that would result in small differences in chord direction.

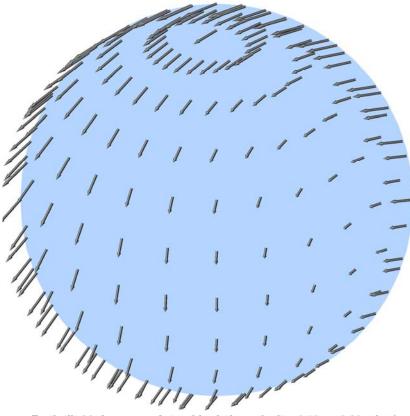**Earth tilt 30 degrees   Asteroid relative velocity 10.00 x, 2.00 y km/s**

Figure 4. Projected relative velocity for a fast asteroid showing little apparent variation in direction across the Earth.

In contrast, Figures 5 and 6 are examples of asteroids with slow apparent motion. In this case there is a striking variation of chord direction depending on observer location. In Figure 5, viewers in the east, south, and west would see the asteroid moving up, across, and down relative to the same star. In Figure 6, slow motion in x and y-directions combine to produce highly variable motion across the Earth's surface. These are very extreme examples chosen to illustrate the potential variety of motions that arise due to an observer's location on the rotating Earth. Although typical occultations won't show this degree of variation in chord direction, the combination of Earth rotation with projection will always play a role in the speed and direction of the chord formed by an asteroid crossing a star.

Earth tilt 30 degrees   Asteroid relative velocity -0.30 x, 0.00 y km/s

Figure 5. Tilted Earth with asteroid moving slowly in x-direction to partially cancel equatorial rotation.

Earth tilt 30 degrees   Asteroid relative velocity -0.50 x, -0.30 y km/s

Figure 6. Tilted Earth with slow asteroid motion in both x, and y-directions.